**Graphing Trigonometric Functions**

1. What is the domain of the function $\sin(x)$? What is its range?

2. What is the domain of the function $\cos(x)$? What is its range?

3. What is the domain of the function $\tan(x)$? What is its range? (This one may be a little more difficult than the first two...)

4. Let's try to graph the sine function. We can start by just plotting some points. Fill in the following table:

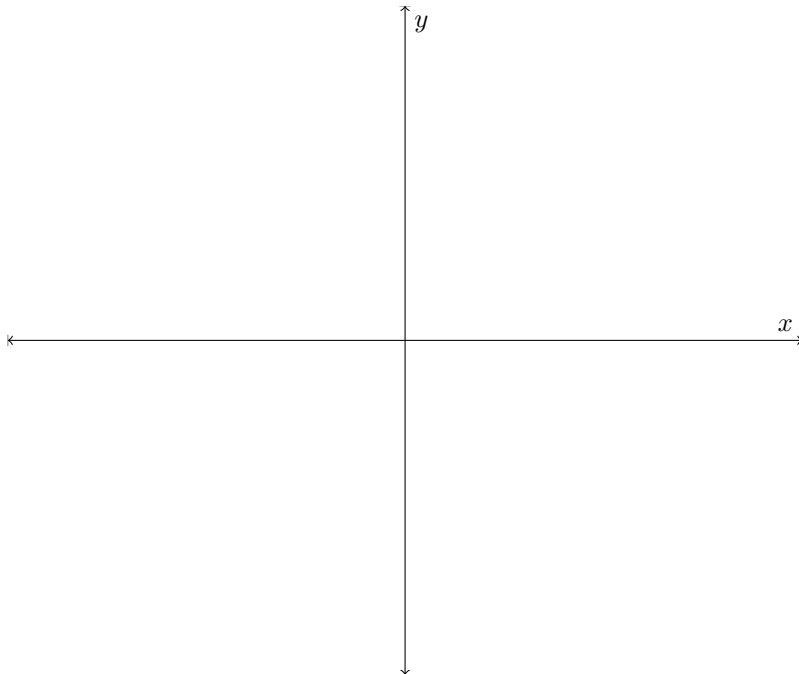| x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin(x)$									

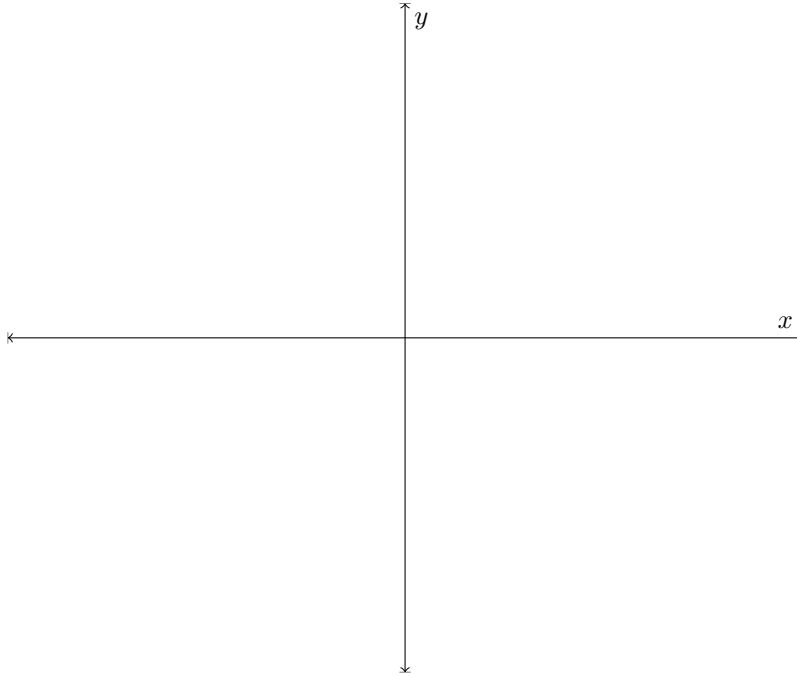5. Plot the points you found in the table.
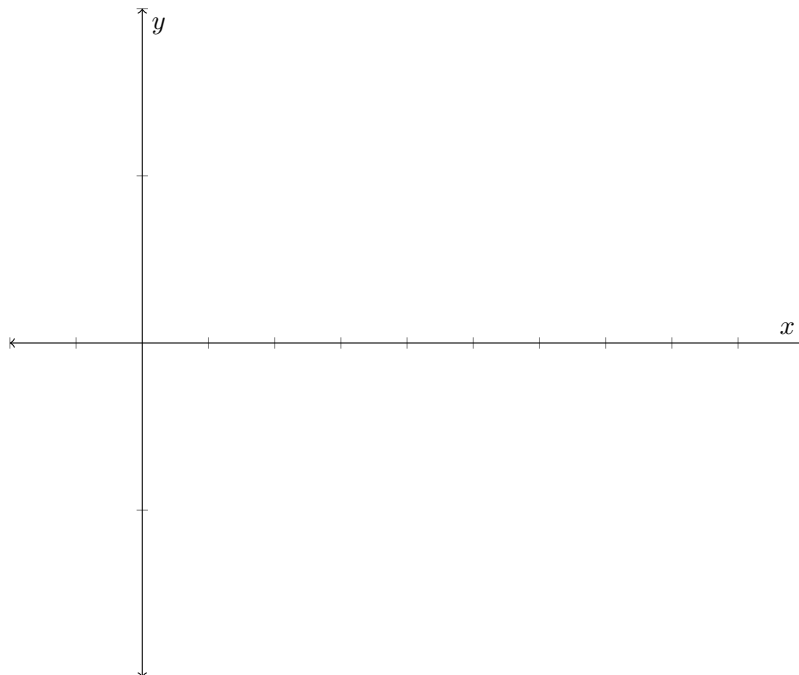
6. Can you fill out the entire graph of $\sin(x)$ from what you found already? Graph the function on the interval $[-2\pi, 4\pi]$. Add appropriate labels.
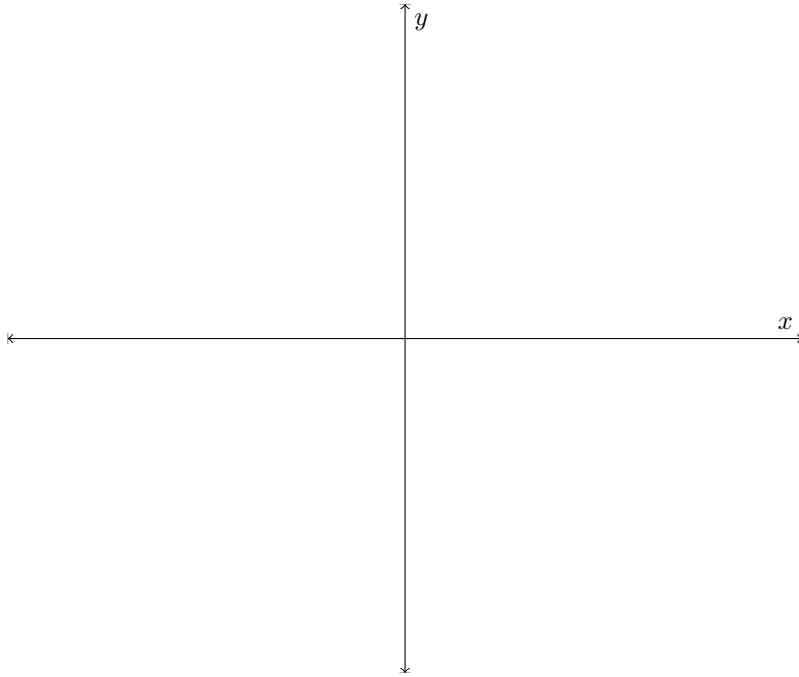
7. Now let's apply some graph transformations. Sketch a graph of $\sin(2x)$. Label carefully.
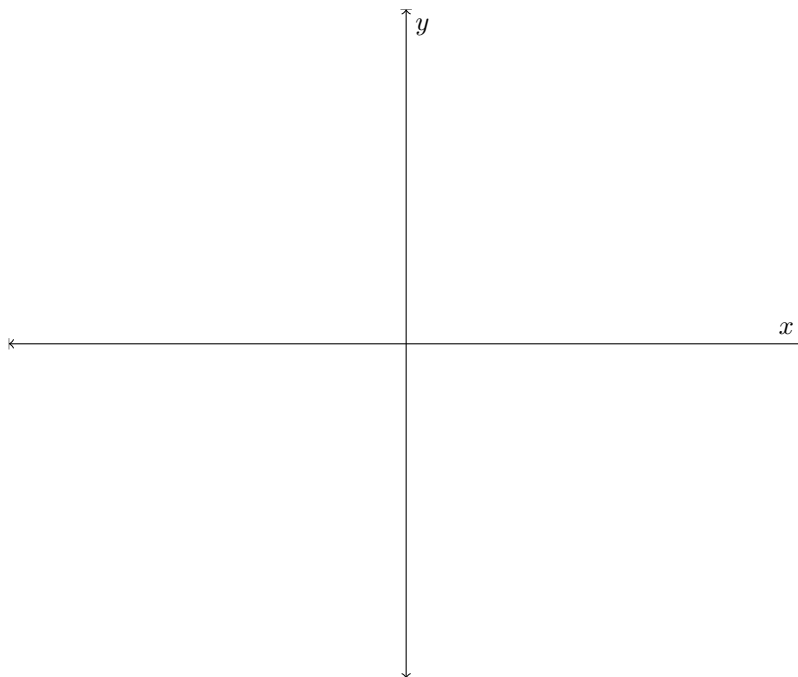
8. Sketch a graph of $2\sin(x - \pi)$. Label carefully.

9. Let's figure out the graph of $\cos(x)$ in a cool way (not just plotting points). We'll use graph transformations.

   (a) What is $\cos(0)$?
   (b) What is $\sin(\frac{\pi}{2})$?
   (c) What is $\cos(\frac{\pi}{6})$?
   (d) What is $\sin(\frac{2\pi}{3})$?
   (e) What is $\cos(\frac{\pi}{4})$?
   (f) What is $\sin(\frac{3\pi}{4})$?
   (g) Can you figure out the pattern? (Hint: what is $\frac{\pi}{6} + \frac{\pi}{2}$?) $\cos(x) = \sin()$.
   (h) Use that last part and graph transformations to graph $\cos(x)$.

10. Now let's graph the tangent function, $\tan(x)$.

  (a) Start by marking all the points where $\tan(x)$ is undefined.

  (b) Now mark all the points where $\tan(x) = 0$.

  (c) The points where $\tan(x)$ are undefined are vertical asymptotes. But which way does the function go, does $f(x) \to \infty$ or $f(x) \to -\infty$? Try to figure it out. Piece together the zeros and the asymptotes to get the graph of $\tan(x)$.
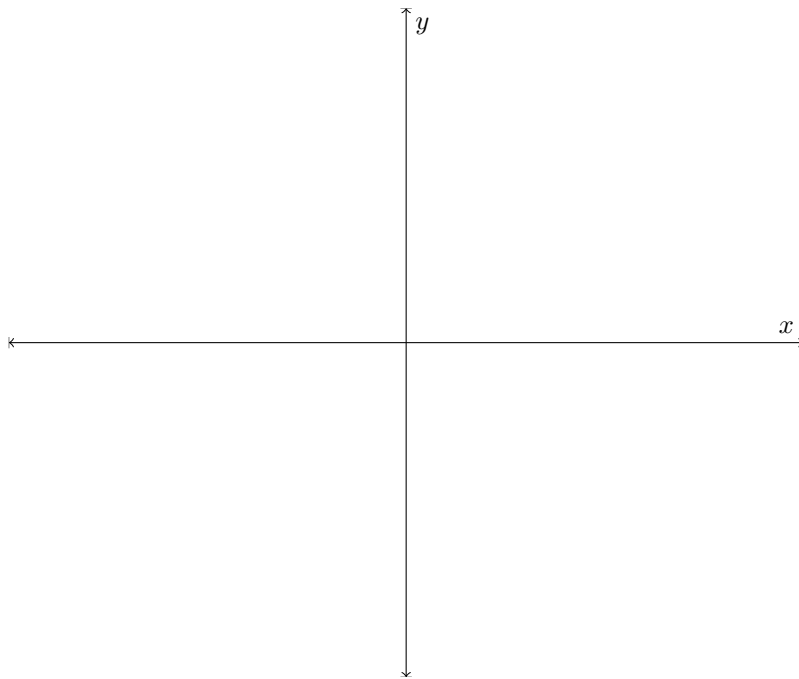
Now you have the basic graphs of the sine, cosine, and tangent functions. As we use these to model periodic behavior, we often want to adjust the *period* and *amplitude* of these functions, and sometimes the *midline*.

11. The *period* of a periodic function is the value a such that $f(x + a) = f(x)$ for all x. That is, it's the length of one iteration of the function before it starts to repeat itself. (The *frequency* of a sine or cosine function is the reciprocal of the period.)

  (a) What is the period of $\sin(x)$?

  (b) What is the period of $\cos(x)$?

  (c) What is the period of $\tan(x)$?

12. The period could change if we modify the functions slightly.

  (a) What is the period of $\sin(3x)$?

  (b) What is the period of $\cos(\frac{x}{2})$?

  (c) What is the period of $\tan(.7x)$?

13. The *midline* of a trigonometric function is the horizontal line that runs through the "middle" of it; that is, the line such that the distance from this line to the maximum value and the distance to the minimum value of the function are equal. For all the basic trig functions, the midline is $y = 0$. However, this can change.

(a) What is the midline of $\sin(x) + 2$?

(b) What is the midline of $\cos(2x - \pi) - 1$?

14. The *amplitude* of a trigonometric function is the distance from the midline to the maximum value.

(a) What is the amplitude of $\sin(x)$?

(b) What is the amplitude of $2\cos(3x) + 4$?

15. Sketch a graph of the function $2\sin(\frac{x}{2} - \frac{\pi}{4}) + 1$. What is its period? Amplitude?
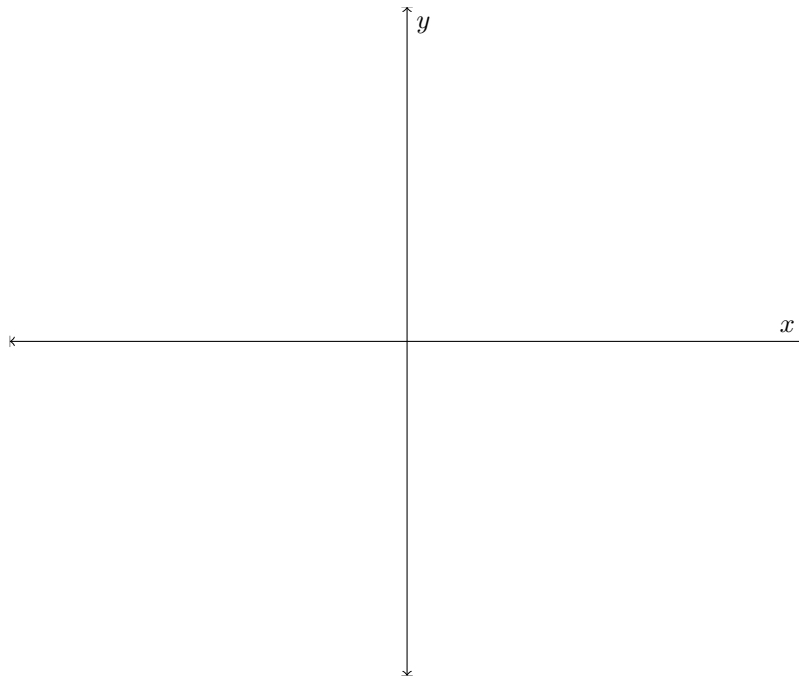

The remaining trig functions are simply the reciprocals of the three we have already graphed. Let's just figure out their basic shapes.
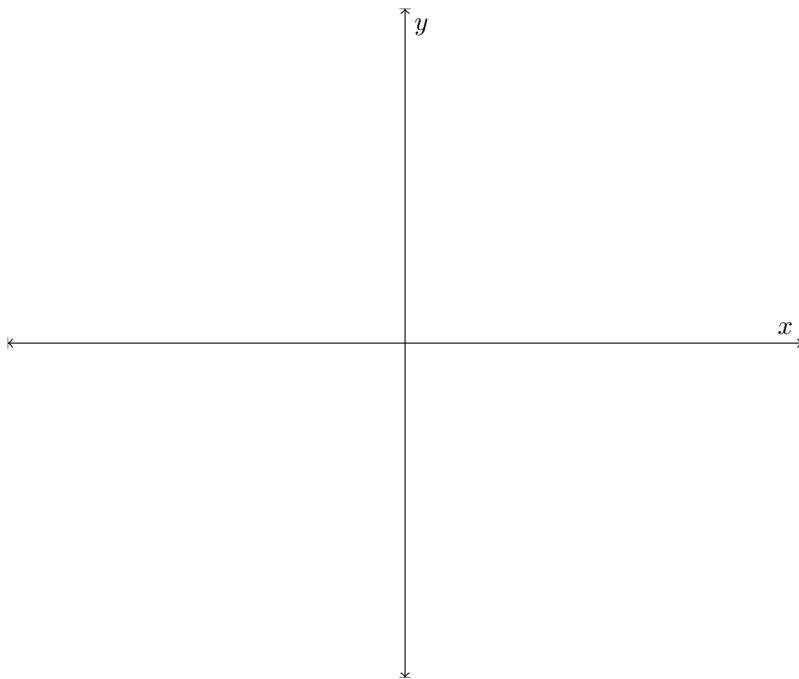
16. If $\sin(x) = 1$, what does $\csc(x)$ equal?

17. If $\sin(x)$ is a small number close to zero, is $\csc(x)$ small or big?

18. If $\sin(x)$ is positive, is $\csc(x)$ positive or negative?

19. If $\sin(x)$ is negative, is $\csc(x)$ positive or negative?

20. Where is $\csc(x)$ undefined?

From that information you just figured out, you can sketch a rough graph of $\csc(x)$.

21. On the axes below, graph both $\sin(x)$ and $\csc(x)$.
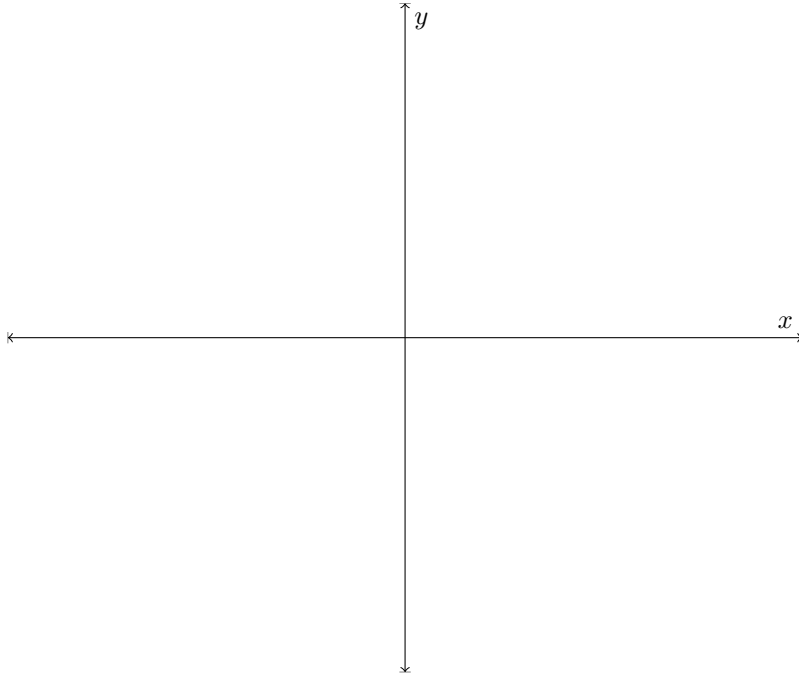
22. Do the same for $\cos(x)$ and $\sec(x)$.

23. Finally, try to sketch a graph of $\cot(x)$.